Autonomous Agents 2: Multi-objective decision problems

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April 30, 2015

Contents

A simple problem...

- Let's say you have a wart on your finger
- Virus, painful, contagious, can in very rare cases lead to skin cancer
- Two treatments:
  - 97.00% probability of being cured
  - 99.99% probability of being cured

Picture by Steven Fruitsmaak, from http://nl.wikipedia.org/wiki/Bestand:Wart_ASA_animated.gif

A slightly more complex problem...

The Dutch government has been attempting to decrease traffic jams in the Randstad, for a number of decades now. Any solution should balance:

- Percentage of traffic jams decreased
- ...

Why Multi-Objective?

Real-world problems just *are* Multi-Objective

And AI can help.
Today: MOMDPs and Multi-agent problems

From MDPs ...

A finite single-objective *Markov decision process* (MDP) is a tuple $\langle S, A, T, R, \mu, \gamma \rangle$ where:

- S is a finite set of *states*,
- A is a finite set of *actions*,
- $T : S \times A \times S \to [0, 1]$ is a *transition function* specifying, for each state, action, and next state, the probability of that next state occurring,
- $R : S \times A \times S \to \Re$ is a *reward function*, specifying, for each state, action, and next state, the expected immediate reward,
- $\mu : S \to [0, 1]$ is a probability distribution over initial states, and
- $\gamma \in [0, 1)$ is a *discount factor* specifying the relative importance of immediate rewards.

... to MOMDPs

A finite single-objective *multi-objective Markov decision process* (MOMDP), with n objectives, is a tuple $\langle S, A, T, R, \mu, \gamma \rangle$ where:

- S, A, T, μ and γ are the same as in an MDP, but
- $R : S \times A \times S \rightarrow \Re^n$ is a *reward function*, specifying, for each state, action, and next state, the expected immediate vector-valued reward.

MOMDP equations

MOMDP
$$\mathbf{R}_t = \sum_{k=0}^{\infty} \gamma^k \mathbf{r}_{t+k+1}$$ $$\mathbf{V}^\pi(s) = E[\mathbf{R}_t \mid \pi, s_t = s]$$ $$\mathbf{V}^\pi(s) = \sum_a \pi(s,a) \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \mathbf{V}^\pi(s')]$$

Additive vector valued returns.

But wait...

Can't we just *scalarize* the decision problem?

- Find a function f that translates the multiple objectives to a scalar utility: a scalarization function

$$V_{\mathbf{w}}^{\pi} = f(\mathbf{V}^{\pi}, \mathbf{w})$$

- Use the scalarization function to define an equivalent single-objective problem
- Solve that problem, and we're done (`http://incompleteideas.net/rlai.cs.ualberta.ca/RLAI/rewardhypothesis.html`)
- ... right?

Scenarios

- We identified 3 scenarios in which scalarization before planning or learning is:
  - Impossible
  - Undesirable
  - Infeasable
- These scenarios are called:
  - (a) unknown weights
  - (b) decision support
  - (c) known weights

The unknown weights scenario

- We know the scalarization function, but not the weights $\mathbf{w}$, for an MOMDP.
- Planning (or learning), is expensive, but we have quite a bit of time before we need to act.
- When the weights come in however, we want to act immediate.
- Furthermore, the weights may change quickly.
- Example: resources and costs of varying prices on the market and mining company trying to obtain resources.

Decision support scenario

- We know the MOMDP (or might have a simulator), but it is hard to construct a plan for it.
- Furthermore, the trade-offs between the objectives are hard.
- The people who have to determine the weigths, e.g. a committee at a local government, want to be presented with all the alternatives first.

- Example: Changing the traffic situation of a city to improve the flow of traffic, while minimizing noise levels and pollution.

Multiple policies

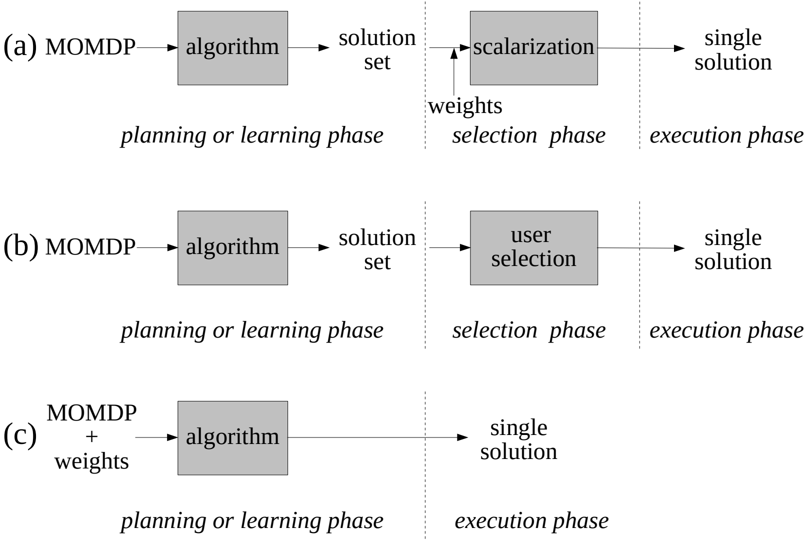
- The *unknown weights* and *decision support* scenarios, require a solution for all *possible scalarizations*.
- They require *multiple policies* to be computed.

Known weights scenario

- We know the MOMDP (or might have a simulator), and know the scalarization function, and weights, but still cannot scalarize.
- The form of the scalarization function is complex.
- If we try to scalarize the problem before planning or learning, this can lead to undesirably complex scalar valued problems.

- The known weights scenario is a single-policy scenario, but does require special methods.

Scenarios


(a) MOMDP
algorithm
solution set
scalarization
weights
single solution
planning or learning phase
selection phase
execution phase
(b) MOMDP
algorithm
solution set
user selection
single solution
planning or learning phase
selection phase
execution phase
(c) MOMDP + weights
algorithm
single solution
planning or learning phase
execution phase

Contents

Taxonomy

- So, let's solve some MOMDPs
- We have seen that:

$$V_{\mathbf{w}}^{\pi} = f(\mathbf{V}^{\pi}, \mathbf{w})$$

- Solving for all possible scalarizations gives us the undominated set:

$$U = \{\pi : \exists \mathbf{w} \forall \pi' \ V_{\mathbf{w}}^{\pi} \geq V_{\mathbf{w}}^{\pi'}\}$$

- But what is the scalarization function?
- What type of policies do we require/allow?

Taxonomy

- So, let's solve some MOMDPs
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- But what is the scalarization function?
  $\rightarrow$ a property of the problem!
- What type of policies do we require/allow?
  $\rightarrow$ a property of the problem!

MOMDP Taxonomy

- type of scalarization function (linear, monotonically increasing)
- stochastic policies versus deterministic policies
- single policy versus multiple policies

Scalarization functions

- Linear scalarization function:

$$V_{\mathbf{w}}^{\pi} = f(\mathbf{V}^{\pi}, \mathbf{w}) = \mathbf{w} \cdot \mathbf{V}^{\pi}$$

- Some function that is *monotonically increasing* in all objectives

The linear scalarization function

- Linear scalarization function:

$$V_{\mathbf{w}}^{\pi} = f(\mathbf{V}^{\pi}, \mathbf{w}) = \mathbf{w} \cdot \mathbf{V}^{\pi}$$

- Common, e.g. prices of different resources
- What happens if we the scalarization function is linear and we know the weights?

The linear scalarization function

- Linear scalarization function:

$$V_{\mathbf{w}}^{\pi} = f(\mathbf{V}^{\pi}, \mathbf{w}) = \mathbf{w} \cdot \mathbf{V}^{\pi}$$

- Common, e.g. prices of different resources
- Known weights scenario $\rightarrow$ trivial (single objective MDP translation)
- Undominated set $\rightarrow$ Convex Hull

$$CH = \{\pi : \exists \mathbf{w} \forall \pi' \ \mathbf{w} \cdot \mathbf{V}^{\pi} \geq \mathbf{w} \cdot \mathbf{V}^{\pi'}\}$$

The linear scalarization function

- We need only to consider stationary deterministic policies.
- Why?

The linear scalarization function

- We need only to consider stationary deterministic policies.
- Why? Hint:
  - For single objective MDPs we know that there always is an optimal policy (one with the maximum possible value), that is deterministic and stationary...
  - ...

The linear scalarization function

- We need only to consider stationary deterministic policies.
- Why?
  - For single objective MDPs we know that there always is an optimal policy (one with the maximum possible value), that is deterministic and stationary.
  - For each possible weight vector $\mathbf{w}$, an MOMDP with a linear scalarization problem can be translated to an equivalent MDP
  - Ergo, for all possible weight vectors, there is an optimal policy that is deterministic and stationary.

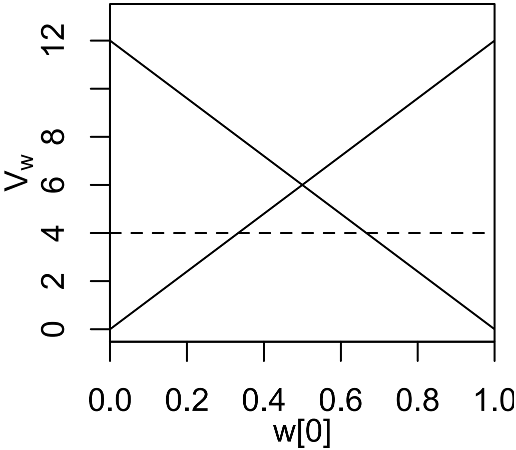
Exercise

- Three armed bandit (single state), with deterministic direct (2 dimensional) rewards:
- $a_1 \rightarrow (3, 0)$
- $a_2 \rightarrow (1, 1)$
- $a_3 \rightarrow (0, 3)$
- $\gamma = 3/4$
- Find the Convex Hull

$$CH = \{\pi : \exists \mathbf{w} \forall \pi' \ \mathbf{w} \cdot \mathbf{V}^{\pi} \geq \mathbf{w} \cdot \mathbf{V}^{\pi'}\}$$

- What are the (2D) Value(s/ vectors)?

Convex upper surface



Summary Linear Scalarizations

- Single policy
  - One stationary deterministic policy
- Multiple policies
  - Convex Hull of stationary deterministic policies

Monotonically increasing scalarization functions

- What if all we can assume about the scalarization function is that is monotonically increasing in all objectives?
  - If we keep the values for all objectives but one the same, and increase the value of the other one, the scalarized value cannot go down.
- A Pareto-dominated policy is always worse than a non-dominated policy:

$$\mathbf{V}^{\pi} \succ_P \mathbf{V}^{\pi'} \Leftrightarrow \forall i, V_i^{\pi} \geq V_i^{\pi'} \wedge \exists i, V_i^{\pi} > V_i^{\pi'}$$

- Scalarized(!) returns can become non-additive: e.g. if $f(\mathbf{V}^{\pi}, w) = w \prod_i \max(0, V_i^{\pi})$ then,

$$f(\mathbf{V}^{\pi}, w) \neq E[\sum_{k=0}^{\infty} \gamma^k (f(\mathbf{r}_{t+k+1}), w)].$$

Exercise revisited

- Three armed bandit (single state), with deterministic direct (2 dimensional) rewards:
- $a_1 \rightarrow (3, 0)$
- $a_2 \rightarrow (1, 1)$
- $a_3 \rightarrow (0, 3)$
- $\gamma = 3/4$
- The scalarization function is: $f(\mathbf{V}^\pi, w) = w \prod_i \max(0, V_i^\pi)$ if all , where w is a positive constant.
- What is the optimal policy?

Exercise revisited

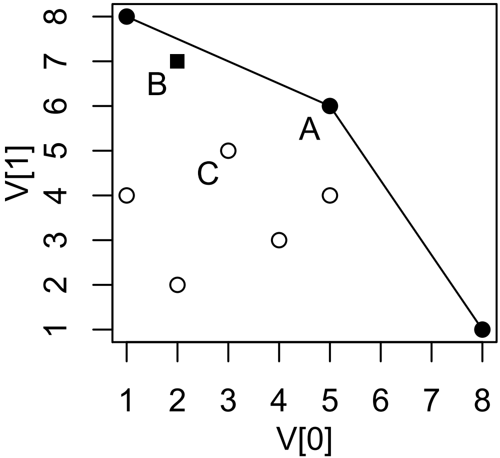
- Three armed bandit (single state), with deterministic direct (2 dimensional) rewards:
- $a_1 \rightarrow (3, 0)$
- $a_2 \rightarrow (1, 1)$
- $a_3 \rightarrow (0, 3)$
- $\gamma = 3/4$
- The scalarization function is: $f(\mathbf{V}^{\pi}, w) = w \prod_i \max(0, V_i^{\pi})$, where w is a constant.
- What is the optimal policy?
  - Stochastic policy?
  - If we allow only deterministic policies, can we suffice with stationary policies?

Monotonically increasing scalarization functions

- If we do not allow stochastic policies,
  - E.g. in medical decisions it is not exceptable to take actions stochastically
  - It is not acceptable to treat patients stochastically and look only at the *expected* (read average) returns.
- We may have to resort to non-stationary policies, i.e. policies that condition their actions on time.
- This does not occur in single objective MDPs!
- We need the Pareto-front of deterministic non-stationary policies:

$$PF = \{\pi : \neg\exists\pi', \mathbf{V}^{\pi'} \succ_P \mathbf{V}^{\pi}\}.$$

Pareto front vs. convex hull



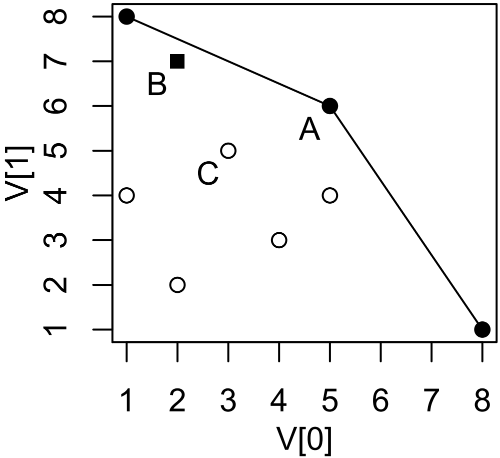
Monotonically increasing scalarization functions

- If we do allow stochastic policies,
  - There is a nice trick to combine 2 or more stationary deterministic Convex Hull policies, to the optimal undominated policies:
  - Consider a mixture policy: stochastically select one of the deterministic policies to follow.
  - e.g. a mixture policy π_m, of a policy π_1 with value $(3, 0)$, and another policy π_2 with value $(0, 3)$, will yeild the following value:

$$\mathbf{V}^{\pi_m} = p_1 \mathbf{V}^{\pi_1} + (1 - p_1)\mathbf{V}^{\pi_2} = \left(\frac{3p_1}{1-\gamma}, \frac{3(1-p_1)}{1-\gamma} \right)$$

depending on the value of p_1.

Values of mixture policies?



Monotonically increasing scalarization: single policy

- One stochastic and/or non-stationary policy
- If stochasticity is allowed: one mixture policy

Monotonically increasing scalarization: summary

- Stochastic policies: convex hull of deterministic stationary policies + mixture policies
- Deterministic policies: Pareto front of deterministic non-stationary policies

Summary

	single policy (known weights)		*multiple policies (unknown weights or decision support)*	
	deterministic	stochastic	deterministic	stochastic
linear scalarization	one deterministic stationary policy (1)		convex hull of deterministic stationary policies (2)	
monotonically increasing scalarization	one deterministic non-stationary policy (3)	one mixture policy of two or more deterministic stationary policies (4)	Pareto front of deterministic non-stationary policies (5)	convex hull of deterministic stationary policies (6)

Methods: inner loop versus outer loop

- Inner loop
  - Perform a series of multi-objective operations (e.g. Bellman backups)
  - Typically by adapting operators of a single objective method (e.g., value iteration)
- Outer loop
  - Use a single objective method as a subroutine
  - Solve as a series of single-objective problems

(Inner loop) Methods

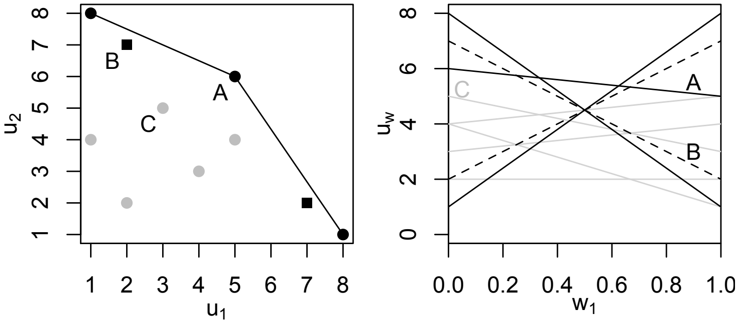
- Planning Multiple policies
  - Convex Hull value iteration (Barret and Narayanan (2008)
  - CON-MOMDP (deterministic stationary Pareto Front) (Wiering and De Jong (2007))
  - (hypervolume) Monte-Carlo Tree Search (PF) (Wang and Sebag (2012))
  - Various LP methods
- Learning
  - Model-based (Lizotte et al. (2010) and Lizotte et al. (2012))
  - Policy search (e.g. Policy gradient (SP), evolutionary methods (PF))
  - TD methods (e.g. Hiraoka et al. (2009), Mukai et al. (2012))
  - Pareto Q-learning (Van Moffaert and Nowé (2014)) (model-based?)

Contents

Optimistic Linear Support

- Outer loop for finding a CCS (lossless subset of the CH)
- Generic multi-objective method
- Repeatly calls a single-objective solver
- Inherits quality bounds from single-objective method

Relation with POMDPs



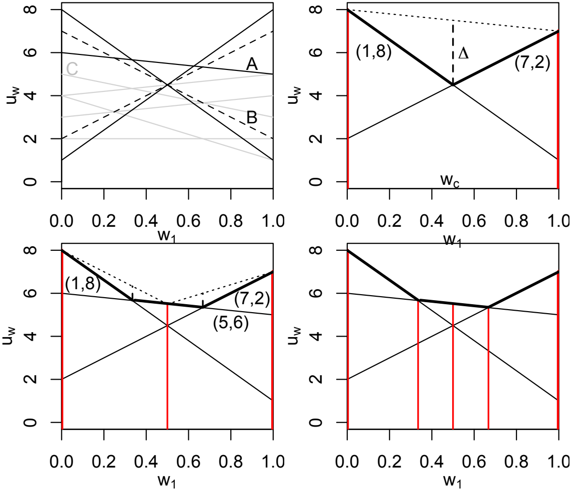
Piece-wise linear and convex scalarized value function:

$$V^*_{CCS}(\mathbf{w}) = \max_{\mathbf{V}^\pi \in CCS} \mathbf{w} \cdot \mathbf{V}^\pi$$

Linear Support

- Algorithm from POMDP literature
- Can be adapted to multi-objective setting
  - Beliefs $\rightarrow$ weight vectors
  - α-vectors $\rightarrow$ value vectors
- Find the CCS without enumerating all policies, by solving scalarized instances at specific weight vectors $\mathbf{w}$

Linear Support

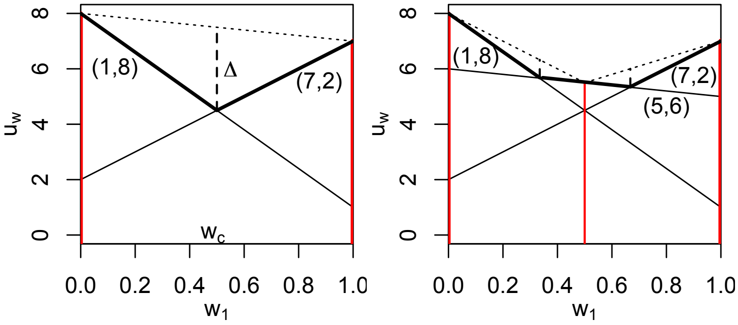


Linear Support

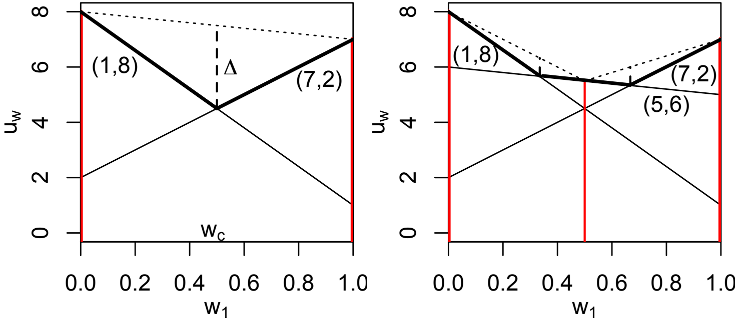
Find the CCS without enumerating all policies

1. Start with an empty set of value vectors S
2. Put the extrema of the weight simplex, $(0, 1)$ and $(1, 0)$, on a queue Q
3. While Q is not empty
   - Solve scalarized instances at every weight vector in Q
   - Add solutions to S
   - Calculate new corner weights (where the values intersect) and add them to Q

Optimistic CCS

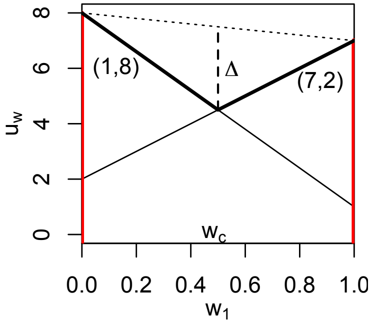


Optimistic CCS $\rightarrow$ Optimistic Linear Support



Use a priority queue with Δ as priority.

ε-CCSs

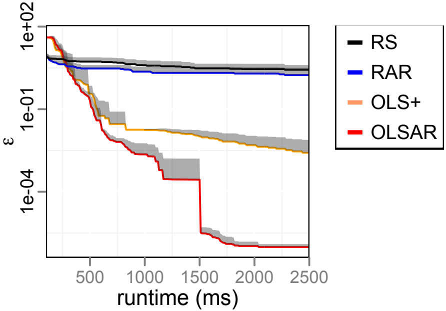


Theorem

During execution of OLS, S is an ε-CCS with $\varepsilon \leq \Delta(\mathbf{w}_1)$, where $\mathbf{w}_1$ is the corner weight with the highest priority in Q.

Reusing value functions found at earlier iterations

- Observation: when two weights are close, the scalarized values are probably close
- Observation: if we can just check whether the value at a given weight is still optimal, we might be done immediately
- Optimistic Linear Support with Alpha Reuse (OLSAR) for MOPOMDPs



OLS

- OLS is a generic multi-objective method
  - MOMDPs
  - MOPOMDPs
  - Multi-objective coordination graphs
- Can produce the CCS without enumerating all possible policies
- Can produce an ε-CCS (much faster)
- Works with any exact single-objective solver
- Extension for approximate solver (AOLS)
- Reusing value functions from earlier iterations makes OLS much faster (OLSAR)

Concluding

- Thank you for your attention,
- If you are interested in doing a project and/or master thesis on the subject of multi-objective decision problems, please contact us.

Further reading

- MOMDPs:
  - Diederik M. Roijers, Peter Vamplew, Shimon Whiteson, and Richard Dazeley — A Survey of Multi-Objective Sequential Decision-Making. *Journal of Artificial Intelligence Research*, 48:67-113, 2013.
- OLS
  - Diederik M. Roijers, Shimon Whiteson, and Frans Oliehoek. — Computing Convex Coverage Sets for Faster Multi-objective Coordination. *Journal of Artificial Intelligence Research*, 52:399-443, 2015.
  - Diederik M. Roijers, Shimon Whiteson, and Frans Oliehoek — Point-Based Planning for Multi-Objective POMDPs. *International Joint Conference on Artificial Intelligence (IJCAI)*, 2015. To Appear.